

Development
of
ULTRASONIC WELDING EQUIPMENT
for
REFRACTORY METALS

by
Nicholas Maropis

AEROPROJECTS INCORPORATED

West Chester, Pennsylvania

Contract: AF 33(600)-43026

ASD Project No. 7-888

Interim Technical Progress Report

May through July 1962

Pursuant to requirements delineated during Phase I for a heavy-duty ultrasonic welding system, and in conformance with objectives established for Phase II, work has advanced toward the development, design, and construction of a 25-kilowatt spot-type welding unit and associated power source. The design of the welded framework has been completed, in addition to various welding control circuits. Evaluations have continued in isolating and resolving problems relating to critical components of the 25-kilowatt unit, among which are ceramic washer type transducer assemblies, acoustic coupling elements, and power-force programming circuitry. Weldment and tip material investigations continue, with the view of establishing firm data on refractory metal weldability characteristics, and terminal element selection.

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AFSC Aeronautical Systems Division

United States Air Force

Wright-Patterson Air Force Base, Ohio

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ABSTRACT-SUMMARY
Interim Technical Progress Report

ASD Interim Report 7-888(IV)
August, 1962

DEVELOPMENT
OF
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FOR
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Nicholas Maropis
Aeroprojects Incorporated

Pursuant to requirements delineated during Phase I for a heavy-duty ultrasonic welding system, and in conformance with objectives established for Phase II, work has advanced toward the development, design, and construction of a 25-kilowatt spot-type welding unit and associated power source. The design of the welded framework has been completed, in addition to various welding control circuits. Evaluations have continued in isolating and resolving problems relating to critical components of the 25-kilowatt unit, among which are ceramic washer type transducer assemblies, acoustic coupling elements, and power-force programming circuitry.

Weldment and tip material investigations continue, with the view of establishing firm data on refractory metal weldability characteristics, and terminal element selection. ^{SS} ^{NB}

[Materials studied are: D-31 Cl; Inconel X-750; Mo-1/2 Ti; PH15-7Mo; René 41; and W]

Performance studies point to substantiation of Astroloy and Udimet 700 as similarly satisfactory tip alloys for welding refractory metals. ^{NB}

Utilization of ceramic transducer assemblies, under the tension-shell concept, has established, at the lower-power levels, a power-conversion efficiency in excess of projected limits.

The performance of the aluminum-bronze coupler, fabricated during prior study, exceeds that of standard steel units. Performance appears enhanced by sculpturing to provide flexural relief.

Specifications for a 25-kilowatt motor-alternator and associated switching requirements are being established and procurement of the necessary components therefor will follow. ^{all}

[Ultrasonic welding]

FOREWORD

This Interim Technical Progress Report covers the work performed under Contract AF 33(600)-43026 from May 1 through July 31, 1962. It is published for technical information only and does not necessarily represent the recommendations, conclusions, or approval of the Air Force.

This contract with Aeroprojects Incorporated of West Chester, Pennsylvania, was initiated under ASD Manufacturing Technology Project 7-888, "Development of Ultrasonic Welding Equipment for Refractory Metals". It was administered under the direction of Fred Miller of the Fabrication Branch (ASRCTF), Manufacturing Technology Laboratory, AFSC Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio.

This project is under the direction of J. Byron Jones, with Nicholas Maropis serving as Project Engineer. Others associated with this program are Carmine F. DePrisco, Chief Electronics Engineer; John G. Thomas, Metallurgist; Janet Devine, Physicist; Jozef Kozarski, Ultrasonic Welding Laboratory Director; and W. C. Elmore, Consultant. This document has been given the Aeroprojects internal report number of RR-62-47. This is an interim report. The information reported herein is preliminary and subject to further analysis and modification as the work progresses.

The methods used to demonstrate a process or technique on a laboratory scale are inadequate for use in production operations. The objective of the Air Force Manufacturing Methods Program is to develop, on a timely basis, manufacturing process, techniques and equipment for use in economical production of USAF materials and components. This program encompasses the following technical areas:

Rolled Sheet	Powder
Forgings	Component Fabrication
Extrusions	Joining
Castings	Forming
Fiber	Material Removal
Fuels and Lubricants	Solid State Devices
Ceramics and Graphites	Passive Devices
Nonmetallic Structural Materials	Thermionic Devices

Your comments are solicited on the potential utilization of the information contained herein as applied to your present or future production programs. Suggestions concerning additional Manufacturing Methods development required on this or other subjects will be appreciated. Direct any reply concerning the above matter to the attention of Mr. W. W. Dismuke, ASRKRA.

PUBLICATION REVIEW

Approved by:


J. Byron Jones

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT-SUMMARY	3
FOREWORD	4
LIST OF FIGURES	7
LIST OF TABLES	7
INTRODUCTION	8
SECTION:	
<u>I. MATERIAL INVESTIGATIONS</u>	<u>10</u>
Weldment Materials	10
Tip Materials	10
Tip Geometry	12
<u>II. EQUIPMENT DEVELOPMENT</u>	<u>15</u>
Transducer	15
Transducer Design and Test	15
Transducer Evaluation	16
Couplers	19
Geometry	19
Reed	22
Power-Force Programming	23
Power Sources	23
Motor Alternators	23
Switching	27
Structural Details	27
Force System	28
LIST OF REFERENCES	30
APPENDIX A: THE TRANSMISSION OF ULTRASONIC POWER BY FLEXURAL WAVES ON A SLENDER BAR	31
B: CONTACT AREA BETWEEN TWO BODIES HAVING TWO PRINCIPAL RADI	39

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Photomicrographs of Heat-Treated Astroloy and Udimet 700 . .	13
2	Curved Aluminum-Bronze Coupler	20
3	Program Selection Board	24
4	Timing Circuit for Power-Force Programming Unit	24
5	Oscillograms Showing the Response of the Time Base Control Circuit	25
6	Motor Alternator with Power Source and Variable Frequency Transmission	26
7	Instrument and Cabinet Arrangement for 25-Kilowatt Welding Unit	29

LIST OF TABLES

<u>Table</u>		<u>Page</u>
I	Gages and Metallurgical Condition of Weldment Materials On-Order	11
II	Summary of Acoustical Energy Absorber Data for a Nickel and A Ceramic (PZT-4) Transducer Unit of 2-Kilowatt Power- Handling Capacity	17
III	Test Data and Conversion Efficiency of 3.3-Kilowatt Ceramic- Transducer Assembly	18
IV	Evaluation of 4-Kilowatt Wishbone System Welding 2024-T3 Bare Aluminum Alloy	21

DEVELOPMENT
OF
ULTRASONIC WELDING EQUIPMENT
FOR
REFRACTORY METALS
Phase II

INTRODUCTION

Since ultrasonic welding was first demonstrated as a practical method for joining thin gages of aluminum and other common metals and alloys, the equipment capability has been continuously extended to joining materials of increasing thickness as well as newer metals and alloys that are difficult or impossible to weld by other techniques. The aerospace need for high-temperature, corrosion-resistant, refractory metals and alloys has increased the need for ultrasonic welding machines of greater capability than are now available.

The objective of this program is to design, assemble, and evaluate heavy-duty ultrasonic spot- and seam-welding equipment for joining refractory materials and superalloys in thicknesses up to 0.10 inch and to develop necessary techniques for producing reliable welds in these materials. The accomplishment of this objective is divided into three phases: Phase I is concerned with establishing feasibility, defining problem areas, and delineating appropriate solutions thereto; Phase II embraces the development of the required equipment and techniques; and, under Phase III, the performance characteristics of the ultrasonic welding equipment will be demonstrated.

Under Phase I, completed prior to the current reporting period (1)*, the feasibility of producing ultrasonic welds in both monometallic and bimetallic combinations of Cb(D-31), Mo-0.5Ti, Inconel X-750, PH15-7Mo stainless steel, René 41, and tungsten was demonstrated. By extrapolating the weldable gage capability of 4-kilowatt and 8-kilowatt ultrasonic spot-type welders, and utilizing a previously developed first-approximation criterion for the energy required to weld materials of various hardnesses and thicknesses, the electrical power input to the transducer necessary to join the above materials in gages up to 0.10 inch was estimated as approximately 25 kilowatts (2).

* Numbers in parentheses refer to List of References at end of report.

Also under Phase I, the problems involved in the production of heavy-duty ultrasonic welding equipment were delineated, a systematic approach to solving problems was outlined, and design parameters for the requisite heavy-duty spot-welding equipment were defined. The basic concepts involved in such machines were investigated. Spot-type welders for high-power operation were studied in considerable detail. Both theoretical and experimental information were evolved to provide preliminary design requirements for this type of machine.

A survey of the "state of the technology" of transducer materials and coupler materials, supplemented by laboratory investigations, indicated that the transducer-coupling system for the heavy-duty equipment should utilize lead-zirconate-titanate ceramic transducers and aluminum-bronze or beryllium-copper coupling members. The requisite vibratory power can be delivered to the weld zone by means of an opposition-drive transducer-coupling system. Astroloy, a nickel-base alloy made by General Electric Company, was tentatively selected to meet the welding tip material requirements.

The transducer-coupling systems will be driven by a motor alternator providing about 25 kilowatts of electrical power. Solid-state elements may be used to meet the switching requirements.

The work initiated under Phase II has the following objectives:

1. Develop the necessary methods, techniques and equipment to ultrasonically join the selected materials.
2. Design and construct ultrasonic joining unit(s) in accordance with the approach outlined in Phase I.
3. Develop methods and techniques to demonstrate the capability of the equipment developed under Phase II to join the selected materials.

This report describes the work accomplished during the second three months of this phase -- May 1 through July 31, 1962. Emphasis was placed on: investigations related to the properties of the weldment materials, as well as further consideration of welding machine tip materials; the development of the primary equipment elements required in the 25-kilowatt ultrasonic spot-type welding equipment. The third item above, equipment capability studies, will be initiated when the equipment has been assembled.

I. MATERIAL INVESTIGATIONS

"THE OBJECT OF PHASE II IS TO DEVELOP THE NECESSARY METHODS, TECHNIQUES, AND EQUIPMENT TO ULTRASONICALLY JOIN THE SELECTED MATERIALS."

Investigatory work in this area continues to concern itself with the selection, procurement, and test of the highest quality material for use in the program. Such examinations include the response of the materials to ultrasonic welding, and the limited determination of requirements for welding-machine settings which will produce crack-free welds with predictable probability. In addition, investigation is progressing in determining the requirements for sonotrode tips capable of satisfactory performance in energy delivery and extended service life.

WELDMENT MATERIALS

So that performance evaluations of the projected 25-kilowatt ultrasonic spot-welding machine may be made on a scheduled basis, orders were entered for the refractory metals and super-alloys listed in Table 1. Some of these have been received, and are in use in conjunction with the studies proceeding on tip materials.

TIP MATERIALS

So that the selection of a tip material for the terminal element of the 25-kilowatt spot-welding machine could be expedited, additional stocks of Astroloy and Udimet 700 were purchased.

Prior photoelastic investigations (3, 4) indicated that the spherical radius of an ultrasonic spot-welder tip approximately 50 to 100 times the thickness of the weldment sheet adjacent to the tip was satisfactory for welding material in gages of 0.040 to 0.060 inch. A tip radius about 100 times the weldment sheet thickness appeared reasonable for welding in the material gage range of 0.005 to 0.015 inch. Consequently, tip radii of 0.25 and 0.50 inch were selected for use in tip material investigations covering welding of 0.005-inch material. Investigations relative to tip performance and geometry will be extended to Astroloy and Udimet 700 tips of 0.75 and 1.0-inch radii, working with heavier-gage materials.

Consequently, several tips varying in radii from 0.25 to 1.0 inches were fabricated from hot-rolled Astroloy, and from hot-rolled, stress-relieved Udimet 700 (5), for comparative material evaluations.

Table I
GAGES AND METALLURGICAL CONDITION OF
WELDMENT MATERIALS ON-ORDER

Refractory Material	Procurement Source	(inch)	Metallurgical Condition	Quantity (sq. ft)
Columbium (D-31)	E. I. duPont deNemours & Co.	0.040	Vacuum arc cast; stress- relieved (1)	0.9
		.060		.6
		.100		.6
Inconel X-750	Huntington Alloy Products Division	0.040	Deoxidized and annealed	4.0
		.100		2.5
Molybdenum-0.5% Ti	Universal-Cyclops Steel Corp.	0.040	Arc cast; cross-rolled; stress-relieved	1.8
		.060		1.9
		.100		
PH15-7Mo Stainless Steel	Hamilton Watch Co.	0.005	Annealed	5.0
		.010		5.0
		.020		5.0
		.040		5.0
	Peter A. Frasse & Co.	0.032	Condition A	7.5
		.040		7.5
	Armco Steel Inc.	0.090	No. 1 finish, hot-rolled, 30 annealed and pickled	30.0
Rene 41	Alloy Metal Co.	0.020	Annealed and pickled (2)	2.0
		.060		4.5
		.096		2.5
Tungsten	Fansteel Metal- lurgical Corp.	0.020	Powder metallurgy material, cross-rolled and stress- relieved	1.0
		.040		1.1
		.060		1.3
		0.100		0.7

(1) Material ordered cut to specimen size.

(2) Metallurgical condition applies to all gages of individual material.

Examination of the microstructure of these tips, after completion of the heat-treat cycle, showed the two alloys to be nearly identical, and in fact indistinguishable (Figure 1). Since they are fundamentally alike in chemical composition, the performance characteristics of Udimet 700 and Astroloy tips are also anticipated to be similar. Sufficient quantities of bar stock of each alloy exist, so that tips of one or the other may be fabricated for the 25-kilowatt spot-welding machine when investigations of tip material are finalized.

Performance data were obtained for a Udimet 700 tip used in welding thin gages of Cb (D-31), Mo-0.5Ti, and 304 stainless steel (half-hard), at a number of different machine settings.

A total of approximately 1200 welds was made, 100 of which were on 0.010-inch 304 stainless steel, included in the study for control purposes. Weld strengths were tabulated for approximately every twenty-fifth weld.

It was determined that the tip required hand dressing or re-grinding after 20 to 40 welds. The Udimet 700 tip was discarded after approximately 1200 welds.

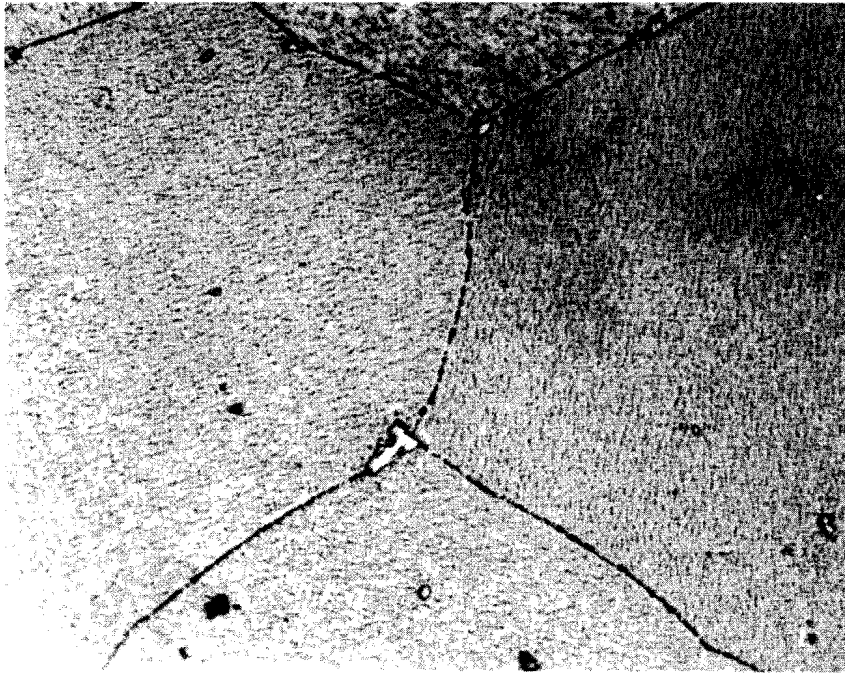
Although substantiation of the contention is required through additional study, it does appear that work to date in this phase has demonstrated that both Astroloy and Udimet 700 are satisfactory as tip alloys for welding the refractory metals. Performance of these materials appears improved after heat treatment.

In these initial tip-performance studies, the anvil surface for the acoustic terminal element was fabricated from Astroloy in the as-cast condition. Occasional surface pitting was observed, and it was necessary to re-position the anvil after every 20 to 40 welds. Surface regrounding was performed after approximately 150 welds. Pitting was particularly noticeable when Mo-0.5Ti was being welded.

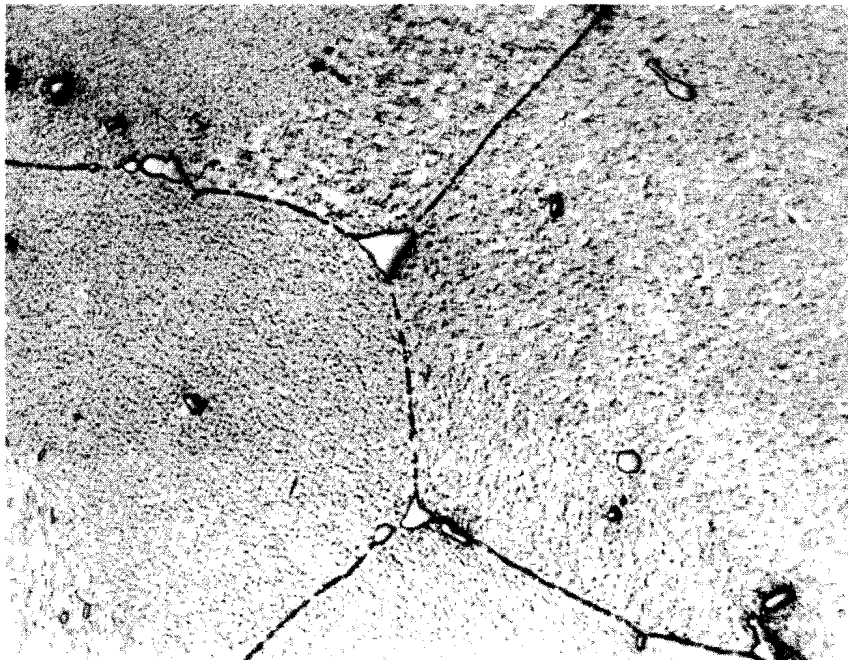
One cast-Astroloy-faced anvil was discarded after approximately 1600 spot-welds, and another after 1200. In an effort to rectify this situation, and to bring service life up to what is regarded as a more satisfactory level, heat-treatable rolled Astroloy plate is being used to make new anvil faces. Following heat treatment, the Astroloy plate will be brazed onto the anvil surface, and evaluations thereof will be made as a part of the tip-material studies.

TIP GEOMETRY

Throughout the evolution of the ultrasonic welding process to the present state of the art, we have almost invariably used a spherical-radius sonotrode tip and a flat passive anvil, except when this combination was unsuitable for work-pieces of a specific geometry.



A. Astroloy



B. Udimet 700

Figure 1

PHOTOMICROGRAPHS OF HEAT-TREATED ASTROLOY AND UDIMET 700

Magnification: 500X
Etchant: $\text{HF} + \text{HNO}_3 + \text{H}_2\text{O}$

A recently developed, high-powered torsional welder yielded more efficient welding, with less deformation of the weldment and a generally more satisfactory weld quality, than is currently realized when spot welding the same materials. The main difference between the torsional welder and the spot welder is that the stress field at the weldment for the former is predominantly one of shear. On this basis, such a stress field will permit more successful welding of the refractory-type materials, and lessen the tendency for cracking which is occasionally observed in spot welding these materials.

A detailed analysis and considerable experimental work is required to confirm or refute this hypothesis. However, if indeed this theory is correct, then a modification of the spot-welding sonotrode tip geometry to increase the in-plane shear stress component, and decrease the in-plane tensile stress component, should largely eliminate whatever residual cracking tendencies remain.

Appendix B is concerned with the geometric changes that can be made to a spot-welding sonotrode tip to achieve this end. The analysis consists of computations of the initial contact area for tips having a range of values for the principal radii, and comparison with initial areas of contact for the case of a spherical tip and a flat anvil.

These data provide reference information for tip designs and confirmatory welding tests.

II. EQUIPMENT DEVELOPMENT

"THE CONTRACTOR SHALL DESIGN AND CONSTRUCT AN ULTRASONIC JOINING UNIT IN ACCORDANCE WITH THE APPROACH OUTLINED IN PHASE I"

Work on the critical components of the 25-kilowatt ultrasonic welding machine has continued, and problem areas are being resolved. Following are details of the work done during this report period:

TRANSDUCER

TRANSDUCER DESIGN AND TEST

Studies oriented to the development of ceramic washer-type transducer assemblies capable of handling 6 to 8 kilowatts of electrical power (as discussed in detail in the first quarterly report of Phase II) were continued.

The 2 kilowatt- and the 3.3-kilowatt units were assembled and tested up to input power levels of 600 and 1250 watts respectively. Difficulties precluded testing to design power levels.

A failure in the acoustical energy absorber, which is used as a reference standard (1) for establishing the performance level of these transducers, resulted in a sudden unloading of the 2-kilowatt unit. Concomitantly, there was a rapid increase in the drive voltage, with arcing in the electrical plug adaptor, and across the ceramic elements.

A rework of the absorber introduced a delay in the transducer development effort. However, since reassembly, testing of the 3.3-kilowatt unit was initiated, and several improvements in the ceramic transducer assemblies have been instituted.

Much of the progress in the development work is based upon measurements taken from the use of the acoustical energy absorber. So that understanding of subsequent tabular data may be facilitated, a re-summary of its operation is hereby given.

The acoustical energy absorber is comprised of a highly absorbing medium for acoustical energy delivered thereto, and copper, water-containing, cooling tubes to carry away that energy which is degraded to heat. Electrical heating elements are spiral-wound over this medium, so that direct-heat energy derived from the electrical power line can be delivered to the medium, and ultimately into the water. The amount of water flow is accurately maintained, as well as the temperature of the input and output water.

At a given flow rate, and water input and output temperature, the electrical power (E_p) supplied to the heating coils corresponds to the acoustical power (A_p) degraded into heat.

A further confirmation is obtained from calorimetric computations of the heat power (H_p) lost to the water. This, too, corresponds to both of the above. Thus $A_p = E_p = H_p$.

The electrical high-frequency input power (I_p) supplied to the transducer is monitored by a high-frequency wattmeter. The conversion efficiency A_p/I_p is then defined by the ratios $\frac{A_p}{I_p} = \frac{E_p}{I_p} = \frac{H_p}{I_p}$.

TRANSDUCER EVALUATION

The first ceramic-driven 2-kilowatt unit, utilizing the tension-shell concept, was assembled and tested by delivering power into the acoustical energy absorber at input levels up to 700 watts. Minor difficulties associated with voltage breakdown in the plug adaptor at the 700-watt level led to arcing adjacent to the ceramic elements, and delayed tests at higher power.

The performance of the ceramic unit, and the power-conversion efficiency realized, are presented in Table II. The projected 60-percent energy-conversion factor discussed in Phase I (see ASD Interim Report 7-888(II), Table 28 and page 99) was exceeded. Actually, a power-conversion efficiency of about 80 percent was achieved.

The 2-kilowatt unit was revised to incorporate improved electrical connections and was performance-tested at higher power levels. The unit will be utilized in actual welding equipment.

The 3.3-kilowatt ceramic transducer was also tested at power levels up to 1250 watts during this period. Table III contains a summary of these data. Electrical driving characteristics were established, and conversion efficiencies were obtained over this range.

These units were designed for non-"heat-limited" operation over their design range. Small radial holes are provided in a central metal washer. Cooling is achieved by passing dry compressed gas through these holes so that the ceramic elements do not overheat. Difficulties were encountered with the 3.3 kilowatt unit resulting from small chip-like Teflon particles from a Teflon insulating sleeve clogging the cooling holes. Subsequent overheating led to failure in the ceramic units at the 1250-watt level.

Corrections have been made to preclude such failures, and an entirely new cooling method utilizing small interconnecting passageways for controlled liquid cooling is contemplated for use on the final 6.6-kilowatt transducer.

Table II

SUMMARY OF ACOUSTICAL ENERGY ABSORBER DATA FOR A NICKEL AND A
CERAMIC (PZT-4) TRANSDUCER UNIT OF 2-KILOWATT POWER-HANDLING CAPACITY

Transducer	Input Power		Water Temperature		Water Flow (gm/sec)	Power Absorbed by Water P_3 (watts)	Transducer Efficiency	
	Transducer P_1 (watts)	To Heater P_2 (watts)	Inflow (°C)	Outflow (°C)			P_2/P_1 (percent)	P_3/P_1 (percent)
Nickel	1000	0	23.5	37.0	6.31	356		36
	0	350	23.5	37.0	6.31		35	
	1650	0	23.5	50.0	5.26	583		35
	0	575	23.5	50.0	5.26		35	
2 KW PZT-4	500	0	23.5	37.5	6.31	370		74
Ceramic	0	370	23.5	37.5	6.31		74	
	600	0	23.4	42.6	6.31	507		84
	0	480	23.4	42.6	6.31		80	
	300	0	23.0	33.7	*	-	-	-
	0	200	23.0	33.7	-	-	67	-

* Water flow was not accurately measured.

Table III
TEST DATA AND CONVERSION EFFICIENCY OF
3.3-KILOWATT CERAMIC-TRANSDUCER ASSEMBLY

Input Power (watts)		Water Temp. (°C)		Water Flow Rate (gm/sec)	Power Absorbed by Water (watts) (P ₃)	Transducer Efficiency (percent)	
To Transducer (P ₁)	To Heaters (P ₂)	Input	Output			P ₂ /P ₁	P ₃ /P ₁
0	200	23	34	4.1			
300	0	23	34	4.1	190	67	63
0	350	23	42	4.1			
500	0	23	42	4.1	330	70	66
0	600	23	57	4.4			
750	0	23	57	4.4	630	80	84
0	925	23	61	6.2			
1000	0	23	61	6.2	980	93	98
0	925	23	62	5.6			
1000	0	23	62	5.6	920	93	92
0	870	23	77	4.2			
1250	0	23	77	4.2	788	75	63*

* System Failed

The ceramic-transducer assemblies as originally conceived and designed remain essentially unchanged. Of the two problems encountered in this development effort, only one was associated with the transducer per se, and that problem is being circumvented by use of a closed-loop liquid-cooling system for the 6.6-kilowatt unit.

The detailed design of the 6.6-kilowatt unit is complete, and the non-critical components will be fabricated during the next quarter.

COUPLERS

GEOMETRY

Consideration of the wedge-reed design for the high-power welder introduced two factors which evolved as the ultrasonic process became better understood, and which were not taken into account in the design of the 2-kilowatt and 4-kilowatt systems.

In a perfectly loaded system - that is, one for which all of the mechanical power supplied from the transducer is absorbed by the load or weldment - the wedge-reed joint should coincide with a flexural antinode on the reed. It has been empirically established, however, that for best transfer of power from the transducer to the reed and ultimately to the weld, the driving point on the reed is displaced from the flexural loop position. The effect of driving off of the loop introduces a flexural component into the driving coupler. Furthermore, if the driving coupler is excessively "stiff", the stress operating at the junction is high. Although this has not been a problem in the 4-kilowatt machines, the stresses at the junction associated with the 12-13-kilowatt power level will lead to unacceptably-low joint life.

During the period covered by this report, a 4-kilowatt coupling system which had been sculptured to provide the flexural relief was tested. The unit was successfully used to weld 2024-T3 structural aluminum alloy. A great deal of ultrasonic welding background information is available on this alloy, in gages of 0.040 and 0.063 inch.

After the performance level was established, the sculptured section was curved, as shown in Figure 2, to simulate the general geometry of the final coupling system.

Evaluation data are given in Table 4. Performance levels of standard 4-kilowatt welding machines are included for reference.

The following conclusions result:

1. The performance capability of the aluminum bronze coupler exceeds that of standard steel units.

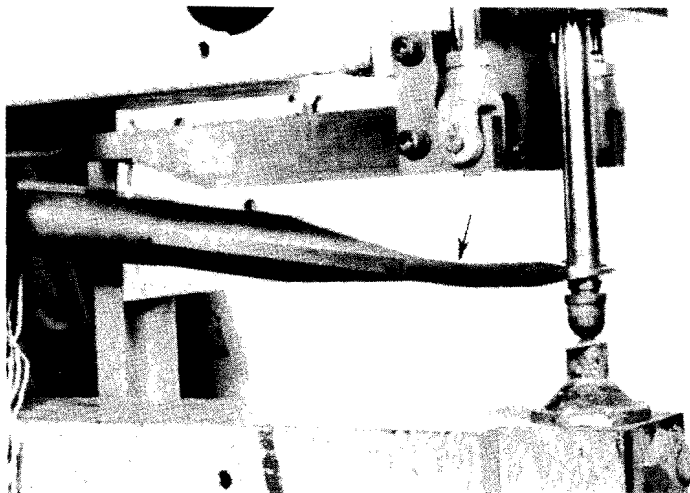


Figure 2

CURVED ALUMINUM-BRONZE COUPLER

(Curved section indicated by arrow - see Figure 14, ASD Interim Report 7-888(II) for significance of this change in coupler geometry.)

Table IV

EVALUATION OF 4-KILOWATT WISHBONE SYSTEM
WELDING 2024-T3 BARE ALUMINUM ALLOY

Systems*	Material Gage, inch	Power, watts	Clamp Force, pounds	Weld Time, seconds	Number of Specimens	Weld Strength			
						Average pounds	Range pounds	St'd Dev. pounds	Deviation percent
A	0.040	2000	900	1.5	100	1030	340-1240	160	16
B		2400	800	1.5	36	950	720-1040	80	9
C		2400	700	1.5	100	950		80	
A	0.050	3700	1000	1.5	20	1050	660-1240	170	16
B		3700	1000	1.5	20	1010	440-1320	260	26
C		3800	1100	1.5	100	1070+133			
A	0.063	4000	1100	1.8	24	1520	950-1750	230	15
B		3700	1100	1.5	8	1010	760-1250	145	15
					3	800+356			
C		3800	1100	1.5	6	930	580-1500		
					3	1030+850			

* System: A - 4-kilowatt straight-wishbone system
 B - 4-kilowatt bent-wishbone system
 C - 4-kilowatt standard system. Typical values obtained by pooling data from several ultrasonic welding systems. Originally published in Aeroprojects Engineering Report No. 12 for Contract No. DA-36-034-ORD-2424, Army Ballistic Missile Agency, Redstone Arsenal, Huntsville, Alabama, July 1958.

2. Sculpturing to provide flexural relief appears to improve performance. Additional work will be required to confirm this.
3. Bending of the coupler through the $4-1/2^\circ$ that simulates the system on the final welder results in a decrease in the performance. However, the level is still acceptable, as the data are superior to that for the reference system. The necessary clearance can be provided in the final unit by bending through only $2-1/2^\circ$. Thus, most of the performance decrease can be recovered.

REED

A theoretical analysis, derived previously (6), considered the characteristic impedance for flexural waves in a slender rod. This analysis served as a guide for the joint design between the wedge-coupling element and the reed. However, the maximum power that can be handled by an ultrasonic system, utilizing the flexural vibratory mode in a coupling element, has not been considered. Since the need was apparent for such an investigation, particularly for defining operating limits, the theoretical study (Appendix A) was made in an effort to (1) establish a power-capability criterion, (2) delineate limits that might exist, and (3) indicate the effect, if any, of geometrical variations on the power-handling capability of the system.

The work indicated that, for equal impedance at a given frequency, a reed of square cross section will transmit 1.3 times as much power as a reed of circular cross section without increasing the associated stresses.

On the basis of this, a 4-kilowatt-capacity square reed and associated supporting mass were designed and released for fabrication. This reed will be incorporated into an existing welding system for confirmatory evaluation.

Design criteria for a reed must take into account its resonant frequency as a free element, the standing wave pattern which exists between the drive point and the mass during power delivery, and the variation of this pattern with time.

These factors will be studied, and any necessary modifications undertaken on the 4-kilowatt system.

A 12-13-kilowatt capacity reed of square cross section was designed for use on the final machine. It is being released for fabrication with excess material in the critical areas, for final adjustment after bench-checking for frequency characteristics.

POWER-FORCE PROGRAMMING

The power-force programming (PFP) circuit is designed to permit operation of the heavy-duty welder with both the power and the force pre-programmed within the selected weld interval. Both of these variables are controlled through a pin-board on which the desired power or force variations with time are preset. Figure 3 depicts one possible combination setting for both power and force. As the unit is designed, the weld interval as set is divided into 10 equal increments so that the level of power and clamping force (each of whose maximum preset value is likewise divided in 10 equal increments) can be adjusted to vary in 10 stepwise increments during the weld cycle.

During this report period the control circuitry components were assembled, wired, and the response at several time increments determined. Figure 4 shows the timing circuit assembly for the power-force programming unit.

Figure 5 contains oscillograms taken at the maximum and minimum time intervals for the circuit.

Each step in Figure 5A and B represents the voltage at the controlled relay that is activated during the particular interval. In Figure 5B it will be noted that the intervals are not identical. This is due to difference in circuit constants. Variable series resistors are being incorporated for initial precision adjustment, and to provide adjustment to accommodate aging effects for the time-controlling components.

The components associated with the control of the clamping force have been received. These may be incorporated into an existing 4-kilowatt spot welder during the next quarter, and evaluated.

POWER SOURCES

MOTOR ALTERNATORS

The efficacy of using variable-frequency alternators for powering ultrasonic welding equipment has been established.

The necessary frequency control, and frequency-stability requirements under pulse-load conditions for a motor-alternator system powering ultrasonic welding equipment have been determined. This was demonstrated in the successful continuous operation of a 7.5-kilowatt unit in a pilot-plant-type demonstration covering a period of 3-1/2 months.

Figure 6 is a photograph of this unit. In this particular arrangement, the drive motor is on a common line with the alternator.

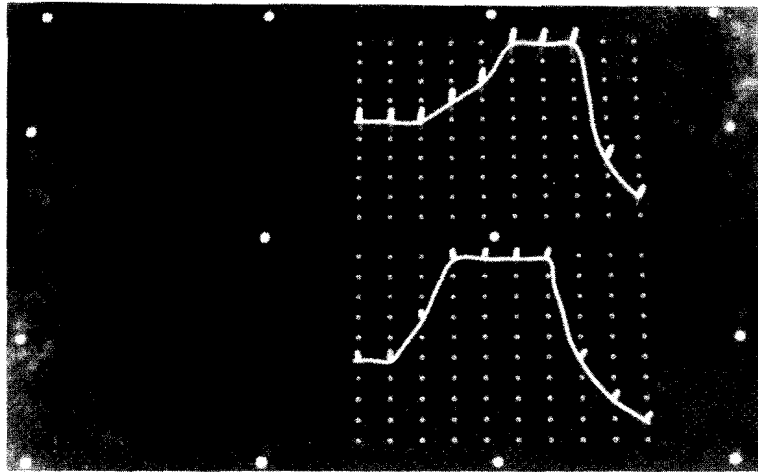


Figure 3

PROGRAM SELECTION BOARD

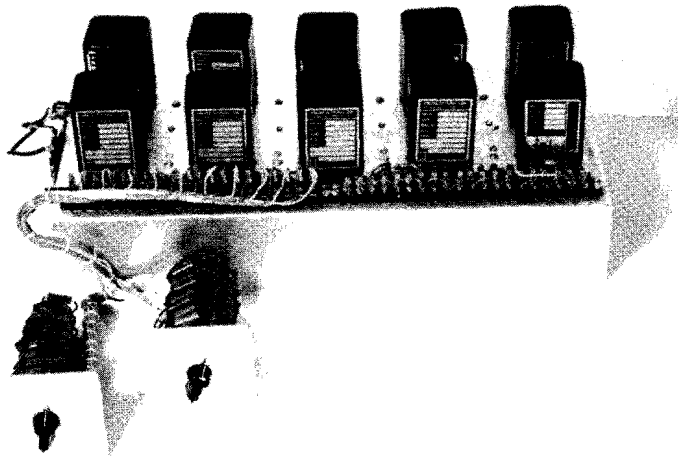
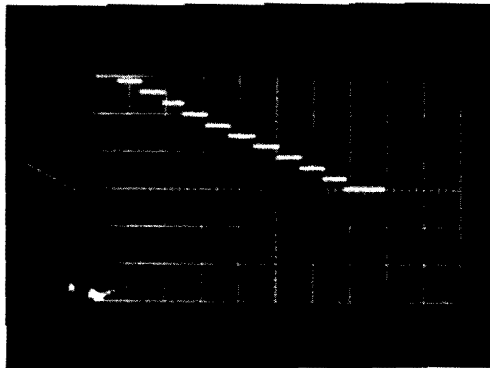
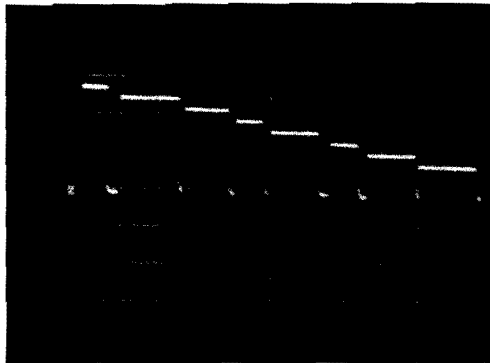


Figure 4

TIMING CIRCUIT FOR POWER-FORCE PROGRAMMING UNIT



A
Total Set Time: 10 secs.



B
Total Set Time: 0.10 sec.
(only 7-1/2 steps shown)

Figure 5
OSCILLOGRAMS SHOWING THE RESPONSE
OF THE TIME BASE CONTROL CIRCUIT

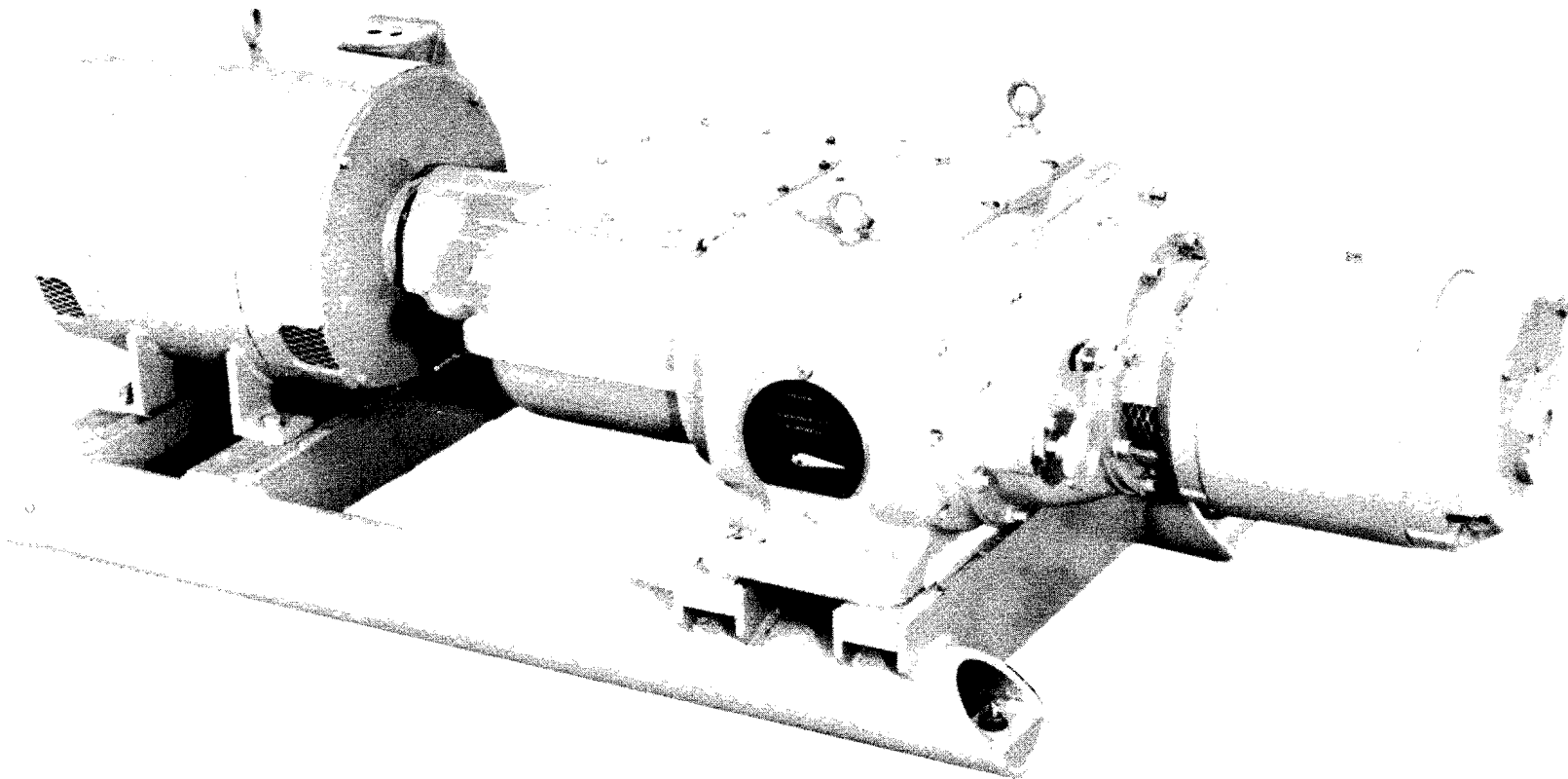


Figure 6

MOTOR ALTERNATOR WITH POWER SOURCE AND VARIABLE FREQUENCY TRANSMISSION

Specifications for the motor-alternator power source for the heavy-duty welder are based upon these data, and are being established in conferences with the various component manufacturers. Cost and delivery information for the large power unit have been obtained, and procurement of the necessary components will follow.

SWITCHING

The details associated with the switching requirements were covered in the first quarterly progress report. Specifically, it is necessary to switch up to the full output power of the alternator, and also to switch dummy resistors into and out of the power transmission line to provide the step power variations for power programming.

During this period a series of performance tests was conducted in which switching of both full load, and partial loads, such as will be encountered during step switching for power-force programming, was evaluated.

The initial response of the solid-state switches as established by earlier bench-type tests was reproduced. At full power, the solid-state switches were found to be sensitive to the voltage-current phase relationship existing in the transmission line at the instant switching is initiated. If the phase relationship existing in an open-circuit line is greatly different from zero, triggering of the switch is unreliable.

Switching of partial loads into, or out of, a loaded transmission line is straightforward. Rapid and reliable switching was achieved.

Successful triggering of the solid-state switches from no load to full load was not completely satisfactory. It is possible to achieve more reproducible triggering by maintaining a resistive load on the alternator at all times, and switching from this load to the transducer on the welding machine for welding. However, this arrangement introduces an additional element in the control circuit which we consider undesirable. Accordingly, data on remote-controlled magnetic contactors were reviewed, and it appears that the necessary "on-off" response can be achieved. Magnetic contactors will be incorporated into the machine as the primary switching elements. Switching of the partial loads for power programming requires more rapid response than is generally achieved with magnetic contactors, and solid-state switches may have to be used for this control function.

STRUCTURAL DETAILS

Design of the welded framework for the 25-kilowatt machine, which satisfies the requirements set forth in ASD Interim Report 7-888(II), has been completed. The various welding control circuits have also been completed.

The design of the basic structure and components was independently reviewed by recognized authorities* for compliance with established machine-structure standards for large units. A stress analysis was also carried out to ascertain adequacy of the structure for torsional rigidity and elastic deflection of the beams under the expected maximum clamping forces.

It was shown by this analysis that adequate torsional rigidity is provided. The maximum deflection will probably not exceed 0.010 inch provided the side support tie-pins between the frame and extension beams are rigid. Tapered pins will be utilized, to assure freedom from looseness in this area, and to meet the above criteria.

Figure 7 is a drawing of the general arrangement of the machine, showing also the instruments and cabinet arrangement.

The controls as shown are located for maximum ease of operation. Those controls to the left of center are associated with the various sequential control steps of the machine, while those on the right provide for the ultrasonic power and vibratory power monitoring instruments. The right side is designed to accept standard 12-inch by 18-inch cabinet front panels onto which the necessary control components and instruments will be mounted.

FORCE SYSTEM

The force system for this machine will incorporate a pneumatically driven hydraulic booster system (Miller Flow Power, Melrose Park, Illinois) which will be used to provide a high-speed advance for contacting the work, and to apply the required clamping force.

Variations in the clamping force for programming will be accomplished by varying the position of an underlapped servo valve.

* Stulen Engineering Co.

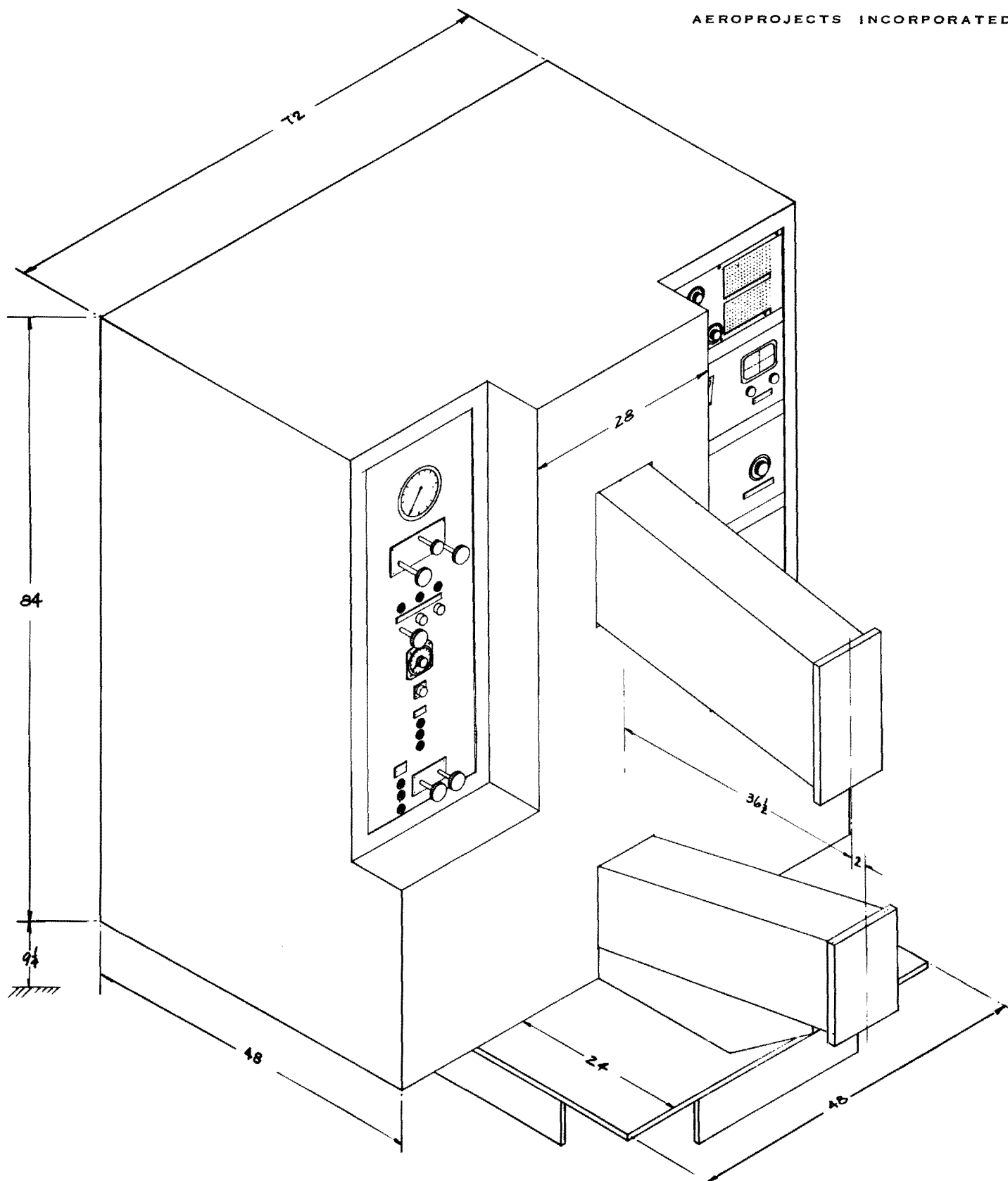


Figure 7

INSTRUMENT AND CABINET ARRANGEMENT FOR 25-KILOWATT WELDING UNIT

LIST OF REFERENCES

1. Aeroprojects Incorporated, "Development of Ultrasonic Welding Equipment for Refractory Metals," ASD Interim Report 7-888 (II) Contract AF 33(600)-43026, December 1961.
2. Ibid, Section VII.
3. Jones, J. B., N. Maropis, J. G. Thomas, D. Bancroft, "Fundamentals of Ultrasonic Welding, Phase I." Research Report 59-105, Navy Contract NOas 58-108-c, May 1959.
4. Jones, J. B., N. Maropis, J. G. Thomas, and D. Bancroft, "Fundamentals of Ultrasonic Welding, Phase II." Research Report 60-91, Navy Contract NOa(s) 59-6070-c, December 1960.
5. Aeroprojects Incorporated, "Development of Ultrasonic Welding Equipment for Refractory Metals," ASD Interim Report 7-888 (III) Contract AF 33(600)-43026, May 1962.
6. Elmore, W. C., "Characteristic Impedance of Rods Used for Transferring Ultrasonic Power," Research Report 56-14, Aeroprojects Incorporated, Army Contract DA-36-034-ORD-1665, March 1956.

APPENDIX ATHE TRANSMISSION OF ULTRASONIC POWER BY
FLEXURAL WAVES ON A SLENDER BAR

It is shown in a previous study*, Eq. (22), that the characteristic impedance of a uniform slender bar for flexural waves is given by

$$Z_f = A \rho c_l \sqrt{\frac{\omega k}{c_l}} \quad (1)$$

where A is the area of the cross section

ρ is the density

c_l is the bar velocity $(E/\rho)^{1/2}$

$\omega (=2\pi f)$ is the angular frequency

k is the radius of gyration of the cross section about the neutral axis.

The flexural impedance equals the impedance for longitudinal waves $A \rho c_l$ times the dimensionless factor $(\omega k/c_l)^{1/2}$ which depends on frequency, the shape of the section and the bar velocity of sound. The power transmitted by flexural waves going in one direction along the bar is then

$$P = 1/2 Z_f \omega^2 \eta_o^2 \quad (2)$$

where η_o is the peak amplitude of the flexural waves. The factor $1/2$ in Eq. (2) averages the sinusoidal time dependence of the flexural wave, which takes the form

$$\eta = \eta_o \sin(\omega t - Kx) \quad (3)$$

The angular wave number K is given by

$$K = \sqrt{\frac{\omega}{c_l k}} \quad (4)$$

as derived in the original work*, Eq. (19).

* Elmore, W. C., "Characteristic Impedance of Rods Used for Transferring Ultrasonic Power".

In the present report we shall examine how an upper limit to the power that can be transmitted is set by the stress fatigue limit and other material properties of the bar, as well as by certain geometrical factors related to the size and shape of the cross section of the bar. In a previous report by W. C. Elmore, "The Limitation on Amplitude Set by Maximum Strain Energy in Vibrating Systems", the effect of the stress limit on the amplitude of free-free flexural vibrations on an unloaded bar has already been considered. Here we are concerned with power transmission, and how the maximum power level can be reached by proper design of the flexural transmission line.

Let us consider first the relation between maximum surface stress σ and the amplitude η_0 of the flexural wave. The following equations, proved in accounts of beam theory, pertain to this case:

$$\sigma = \frac{M}{Z} = \frac{Mh}{I} = Eh \frac{d^2 \eta}{dx^2} \quad (5)$$

where M is the bending moment at any point x ;

$Z = I/h$ is the so-called section modulus;

h is the distance from the neutral axis to the most distant point on the section;

$I = Ak^2$ is the moment of inertia of the section about the neutral axis;

$d^2 \eta / dx^2$ is the curvature of the neutral section, here caused by the flexural wave.

From Eqs. (3) and (4),

$$\frac{d^2 \eta}{dx^2} = -K^2 \eta = -\frac{\omega}{c_l^2 k} \eta \quad (6)$$

On substituting this value for the curvature into Eq. (5), and disregarding the minus sign having to do with phase,

$$\sigma_{\max} = \frac{Eh\omega}{c_l k} \eta_{\max} \quad (7)$$

This equation may be rewritten to show that the maximum (particle) velocity permitted is

$$\dot{\eta}_{\max} = \omega \eta_{\max} = \frac{k}{h} \cdot \frac{\sigma_{\max}}{\sqrt{E\rho}} \quad (8)$$

which therefore depends on the geometrical factor k/h and the material factor $\sigma_{\max}/\sqrt{E\rho}$. On introducing this maximum particle velocity into Eq. (2) and using Eq. (1) for the impedance,

$$P_{\max} = 1/2 \left[\frac{A k^{5/2}}{h^2} \right] \cdot [\omega^{1/2}] \cdot \left[\frac{\sigma_{\max}^2}{\rho^{1/4} E^{3/4}} \right] \quad (9)$$

Equation (9) shows how the maximum power that can be transmitted depends on a geometrical factor, the frequency and a material factor. The equation can be somewhat misleading, however, in that the three factors are inter-related by the requirement that the dimension of the bar in the plane of the flexural wave (its "depth") must be kept small compared with the wavelength λ . Let us therefore introduce this limitation into Eq. (9) by writing

$$h = \alpha \lambda \quad (10)$$

where α is a pure numeric whose maximum value would appear to be approximately $1/8$, that is, the bar should have a depth no greater than one-quarter of the wavelength of the flexural wave. Since, by Eq. (4),

$$\lambda = \frac{2\pi}{k} = 2\pi \sqrt{\frac{c_l k}{\omega}}, \quad (11)$$

the maximum frequency that can be used with a given bar is given by

$$\omega_{\max} = 4\pi^2 \alpha^2 \frac{c_p}{h} \left(\frac{k}{h} \right) \quad (12)$$

If now we introduce this maximum frequency into Eq. (9),

$$P_{\max} = \pi \alpha A \left(\frac{k}{h} \right)^3 \frac{\sigma_{\max}^2}{\sqrt{\rho E}} \quad (13)$$

which shows that the maximum power that can be transmitted is proportional to the area of the bar, a shape factor $(k/h)^3$ and a material factor $\sigma_{\max}^2 / \sqrt{\rho E}$, which is identical with that limiting power transmission by longitudinal waves. [See, for example, Appendix E, "Fundamentals of Ultrasonic Welding", Eq. (9)]. To reach the maximum set by the material factor, and the geometrical factor (area times shape factor) it is of course necessary to operate at the frequency specified by Eq. (12) with α as large as possible. One may look upon Eq. (12), in fact, as defining the value of α . If a frequency is used less than ω_{\max} , (corresponding, for example to $\alpha < 1/8$) then Eq. (12) must be solved for the smaller value of α to be used in Eq. (13) in computing the upper limit to power transmission for the given flexural transmission line. If we denote by α_{\max} , its highest permissible value, corresponding to the frequency ω_{\max} , then at lower frequencies

$$\alpha = \sqrt{\frac{\omega}{\omega_{\max}}} \alpha_{\max} \quad (14)$$

(It would appear that an experiment should be done to test the assumption that $\alpha_{\max} = 1/8$ is a practical upper limit to the half-depth to wavelength ratio.)

Some practical implications will now be considered. In making use of them one must bear in mind that if a load is not matched to the impedance of this, or any other transmission line, the resultant standing wave pattern will reduce the maximum power that can be delivered to the load. Thus, if a standing wave ratio of 10 exists on the line, the power that can be safely delivered is reduced by a factor of 10. Any attempt to increase power delivery by increasing input power will over-stress the surface regions of the bar, and ultimately lead to fatigue failure.

Bar of Circular Section. Let us first compute P_{\max} for a circular steel rod, one-inch in diameter, for which $\rho = 7.84 \text{ gm/cm}^3$, $c_l = 5.17 \times 10^5 \text{ cm/sec}$. $E = 2.1 \times 10^{12} \text{ dynes/cm}^2$ and $\sigma_{\max} = 10^9 \text{ dynes/cm}^2$ ($\sim 15,000 \text{ psi}$). Then $h = 1.27 \text{ cm}$, $k/h = 1/2$, and assuming that $\alpha = 1/8$,

$$f_{\max} = 2\pi (1/8)^2 \times \frac{5.17 \times 10^5}{1.27} \times 1/2 = 20,000 \text{ cps}$$

The maximum power (with unity standing wave ratio) is

$$\begin{aligned} P_{\max} &= \pi^2/8 (1.27)^2 (1/8) \frac{10^{18}}{\sqrt{7.84 \times 2.1 \times 10^{12}}} \\ &= 6.13 \times 10^{10} \frac{\text{ergs}}{\text{sec}} = 6,130 \text{ watts} \end{aligned}$$

If the line is used at 15,000 cps, the maximum power, by Eq. (14), will be about 86.6 percent of this value, or 5,310 watts.

In comparison, the maximum power that can be transmitted by longitudinal waves, on the same bar by Eq. (9) of Appendix E, "Fundamentals of Ultrasonic Welding", is

$$\begin{aligned} P_{\max} &= 1/2 A \frac{\sigma_m^2}{\sqrt{E \rho}} \\ &= \pi/2 (1.27)^2 \frac{10^{18}}{\sqrt{7.84 \times 2.1 \times 10^{12}}} \\ &= 6.24 \times 10^{11} \text{ ergs/sec} = 62,400 \text{ watts} \end{aligned} \tag{15}$$

which is approximately ten times greater ($32/\pi$). For longitudinal waves the frequency can have any value up to a maximum that makes the radius of the transmission rod an eighth of a wavelength. The maximum frequency is therefore

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{c}{8a} \quad (16)$$

which, for the present example, gives

$$f_{\max} = \frac{5.17 \times 10^5}{8 \times 1.27} = 51,000 \text{ cps.}$$

Bar of Square Section. Next consider a steel bar of square section one inch on a side of the same material. Again $h = 1.27$ cm, but $k = h/\sqrt{3}$ and $A = 4h^2$. Hence

$$\begin{aligned} P_{\max} &= \pi/8 \cdot 4(1.27)^2 \cdot \frac{1}{3\sqrt{3}} \cdot \frac{10^{18}}{\sqrt{7.84 \times 2.1 \times 10^{-12}}} \frac{\text{ergs}}{\text{sec}} \\ &= 12,000 \text{ watts,} \end{aligned}$$

which is nearly twice the power the bar of circular section can accommodate.

The maximum frequency, at which this power can be delivered, is

$$f_{\max} = 2\pi (1/8)^2 \frac{5.17 \times 10^5}{1.27} \frac{1}{\sqrt{3}} = 23,100 \text{ cps}$$

due to the more favorable shape factor. At 15 kc/sec, the maximum power is 80.6 percent of that at f_{\max} , or about 9,700 watts, as compared with 5,300 watts for the bar of circular section. Hence at 15 kc/sec, the increased area and shape factor result in an 83 percent increase in the capacity to transmit power. The impedance of the square bar is 37% greater than that of the circular bar because of the increased area and shape factor.

A square bar whose impedance equals that of a circular bar can transmit 1.3 times as much power without stress overload, when both bars are

operated at the same frequency. For equal impedances at a given frequency, it is found from Eq. (1) that the diameter of the circular bar must be 13.3 percent greater than the thickness of the square bar.

Bar of I Section. As a final example, let us consider the bar having the section shown in Fig. 1

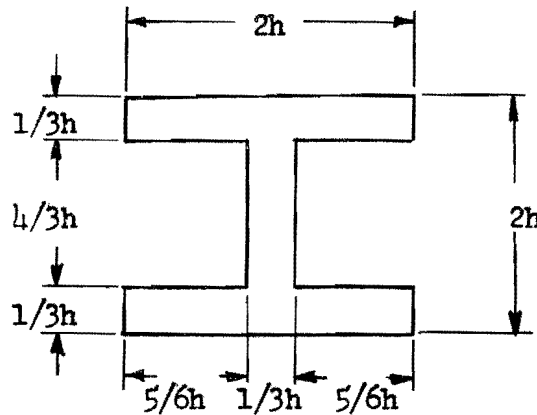


Fig. 1

For this shape it is found that

$$A = \frac{16}{9} h^2, \quad I = \frac{244}{243} h^4, \quad k/h = 0.752$$

Using the same material properties as before and making $h = 1.27$ cm,

Eq. (13) gives

$$P_{\max} = \pi (1/8) \frac{16}{9} (1.27)^2 (.752)^3 \frac{10^{18}}{\sqrt{7.84 \times 2.1 \times 10^{12}}} \frac{\text{ergs}}{\text{sec}}$$

$$= 11,800 \text{ watts,}$$

at the frequency

$$f_{\max} = 2\pi (1/8)^2 \frac{5.17 \times 10^5}{1.27} (.752)$$

$$= 30,000 \text{ cps.}$$

At 15 kc/sec, the maximum power would be 8,350 watts, as compared with 5,300 watts for a circular bar of one-inch diameter and 9,700 watts for a one-inch square bar.

The one-inch I section has an impedance at 15 kc/sec of 1930 kg/sec; the one-inch square bar an impedance of 3800 kg/sec; and the one-inch circular bar an impedance of 2780 kg/sec. The low impedance of the bar of I section means that such a bar can deliver considerably more power, relative to the bars of other sections, when the load has a very low impedance, such as may occur in welding. If the bar of I section has the same impedance as a bar of circular section (which requires that h be 17.5 percent greater than the diameter of the circular bar) the maximum power that can be delivered at the same frequency is 2.3 times the power that can be delivered by the circular bar. It is evident that bars having a high section modulus make the best transmitters of ultrasonic power by flexural waves.

APPENDIX B

CONTACT AREA BETWEEN TWO BODIES HAVING TWO PRINCIPAL RADII

Two elastic bodies forced into contact, and originally having principal radii R_1, R_1' and R_2, R_2' respectively at the contact region, have a contact area that is elliptical with semi-axes given by

$$a = m \left\{ \frac{3\pi}{4} \frac{P (k_1 + k_2)}{A + B} \right\}^{\frac{1}{3}}$$

$$b = n \left\{ \frac{3\pi}{4} \frac{P (k_1 + k_2)}{A + B} \right\}^{\frac{1}{3}}$$

where

P = contact force

$$k = \frac{1 - \sigma^2}{\pi E}$$

σ = Poisson's ratio

E = Young's modulus

A, B are quantities depending on R_1, R_1', R_2, R_2' as defined below,
 m, n are constants depending on A and B , which are found by the
 use of the table.

As shown by Love,

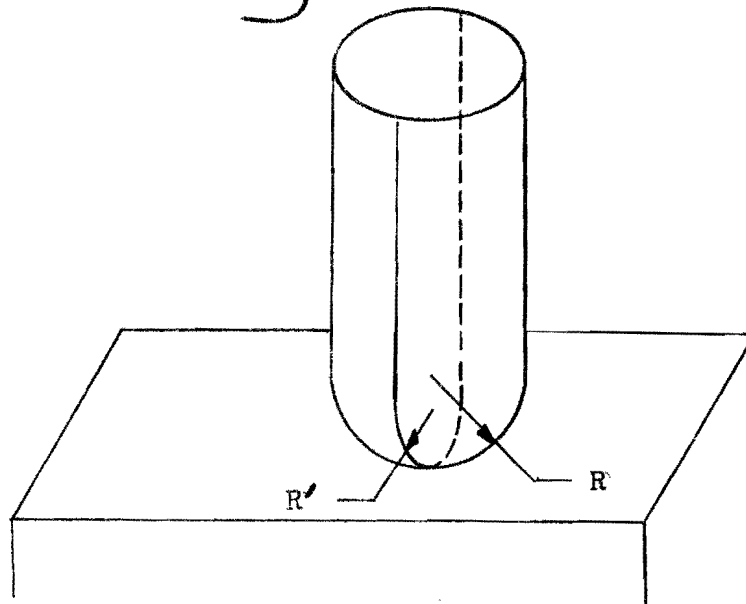
$$2(A + B) = \frac{1}{R} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}$$

$$4(A - B)^2 = \left(\frac{1}{R_1} - \frac{1}{R_1'}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2'}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right)\cos 2\omega$$

where ω = angle between planes containing R_1 and R_2 . We are concerned with the case in which $R_2 = R_2' = \infty$, which is representative of a flat anvil.

Then

$$\left. \begin{aligned} 2(A + B) &= \frac{1}{R_1} + \frac{1}{R_1'} \\ 2(A - B) &= \frac{1}{R_1} - \frac{1}{R_1'} \end{aligned} \right\} \text{Hence} \quad \begin{aligned} A &= \frac{1}{2R_1} = \frac{1}{2R} \\ B &= \frac{1}{2R_1'} = \frac{1}{2R'} \end{aligned}$$



If one defines a parameter θ such that

$$\cos \theta = \frac{B - A}{A + B} = \frac{R - R'}{R + R'}$$

Then m and n are determined from the following table (Timoshenko and Goodier, p. 379).

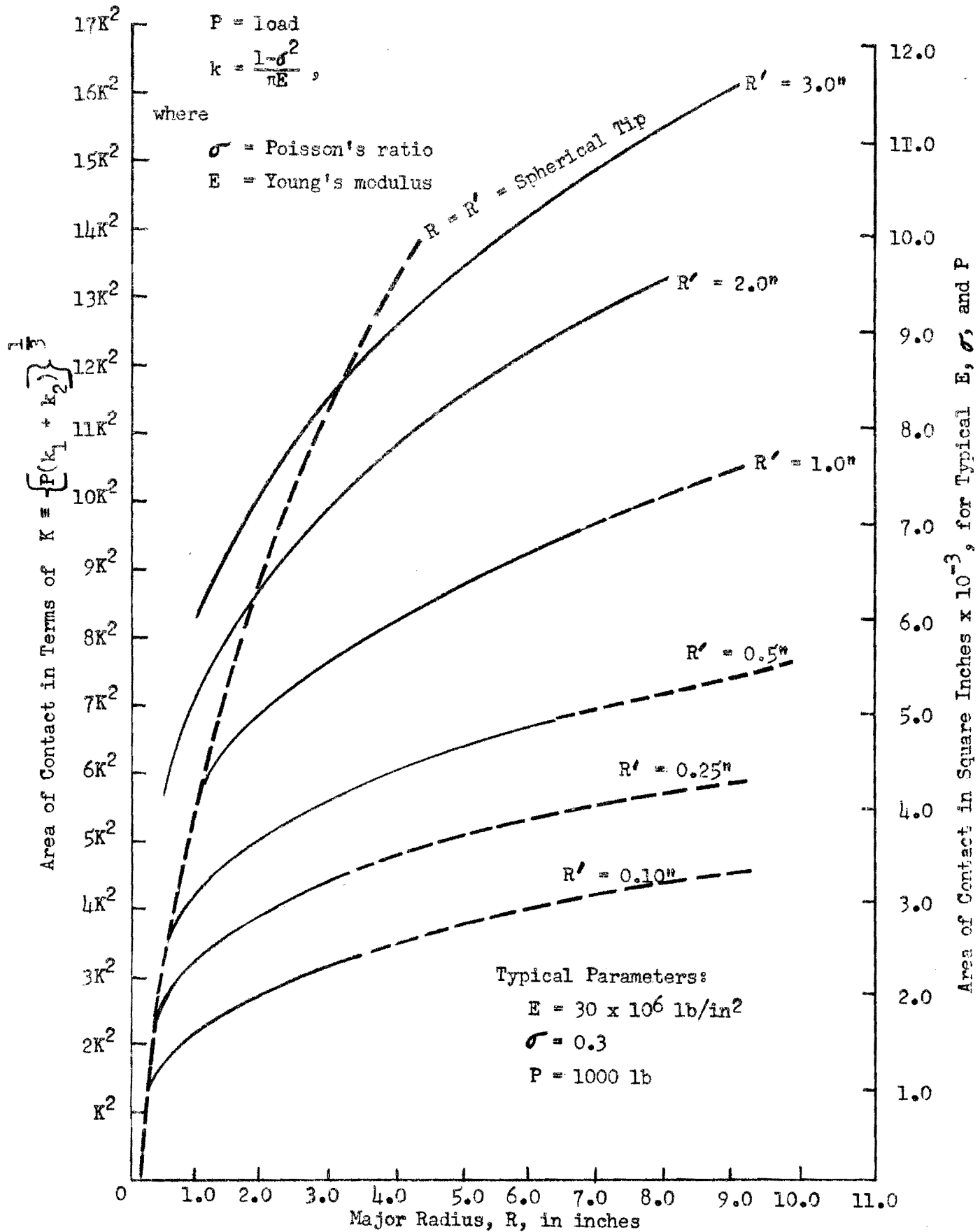
$\theta =$	30°	35	40	45	50	55	60	65	70	75	80	85	90
$m =$	2.731	2.397	2.136	1.926	1.754	1.611	1.486	1.378	1.284	1.202	1.128	1.061	1.000
$n =$	0.493	0.530	0.567	0.604	0.641	0.678	0.717	0.759	0.802	0.846	0.893	0.944	1.000

From the foregoing one may calculate the max. pressure at the center $= \frac{3}{2} \frac{P}{\pi ab}$,
as well as the area of contact $= \pi ab$

The attached figure presents a set of curves relating contact area to the major principal radius for a series of values of the minor radius. The curves show the actual area, for typical elastic constants, and a pressure of 1000 lbs, as well as a scale applying to other elastic constants and pressures.

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1. Love, A Treatise on the Mathematical Theory of Elasticity, p. 192, Dover Publications, New York, 1944.
2. Timoshenko and Goodier, Theory of Elasticity, p. 372, McGraw-Hill Book Company, Inc., 1951.



AREA OF CONTACT AS A FUNCTION OF THE LARGER RADIUS

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